Leakage Flowrate past Pistons of Oil Hydraulic System Components

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The present paper describes the development and use of a relatively simple leakage equation that takes into account the diametral changes of the clearance passage due to the axial pressure drop, and the variation in the oil viscosity during leakage flow due to both the pressure drop and the consequent rise in temperature of the oil. Diametral strain relationships for thick-wall cylinders are utilized to calculate a mean effective clearance for use in the basic-form leakage equation. Nondimensional plots from which diametral strains may be assessed for a wide range of diameters, wall thicknesses, pressures, and materials are included. A simple but fundamentally sound method is given for calculating a mean effective viscosity of the oil during leakage flow through a clearance passage. The method accounts for the effect of pressure and pressure drop on viscosity, together with the effect of temperature rise due to flow. The effectiveness of the proposed leakage flowrate equation is demonstrated by experimental results obtained with pistons having diametral clearances in the range 3 to 20 × 10^{-4} in.

Introduction

Calculation of the leakage flowrate past the piston lands of oil hydraulic system pumps, motors, spool, or other piston-type valves is an important requirement when assessing the quality or performance of such components. The need for accurate leakage calculations increases with the high-pressure, high-precision components used in modern aircraft and spacecraft control systems. It is readily shown that the well-known basic equation for flow in annular clearances is seriously erroneous where high axial pressure drops exist across a piston land. Some designers develop empirical leakage formulas that are satisfactory for certain clearances particular oils, and particular pressure and temperature ranges.

The leakage flowrate through the annular running clearance between the piston and bore of an oil hydraulic pump, motor, or valve is usually calculated with an equation of the form (Fig. 1),[1]

$$Q_x = (13.55 \times 10^{-6} DC^3P'/\mu L)(1 + 1.5\epsilon^2) \text{ in.}^3/\text{min} \quad (1)$$

where piston diameter and land length D and L, respectively, are in inches; diametral clearance C is in 10^{-4} in.; viscosity μ is in centipoises (CP); pressure drop P' is in psi; $\epsilon = 2e/C$ is the eccentricity ratio describing the radial position of the piston in the bore; and e is the radial displacement of the piston and bore axes, 10^{-4} in.

Equation (1) applies only to configurations where the piston and bore axes remain parallel. However, the two parallel axes cases $\epsilon = 0$ (piston concentric with bore) and $\epsilon = 1$ (piston fully eccentric against bore) provide the limiting cases of minimum and maximum leakage flowrate. Equation (1) gives reasonable results if it can be assumed that diametral clearance C is unaffected by pressure, if the axial pressure drop along the piston land can be assumed to be linear, and if oil viscosity μ can be considered as constant during leakage.

It is readily shown that Eq. (1) is inadequate if the pressure drop involved is sufficiently high that both clearance and viscosity are affected by it. The present paper describes readily applied methods for calculating a mean effective clearance, and a mean effective viscosity, which can render Eq. (1) effective for large pressure drop situations. The leakage equation becomes

$$Q_x = (13.55 \times 10^{-6} DC_e^3P'/\mu_e L)(1 + 1.5\epsilon^2) \text{ in.}^3/\text{min} \quad (2)$$

where μ_e is the mean effective viscosity of the oil during leakage flow, centipoise; and C_e is the mean effective diametral clearance, 10^{-4} in.; with D, P', L and ϵ as defined with Eq. (1). Both μ_e and C_e are functions of pressure drop.

Mean Effective Clearance

Pressure inside a cylinder acts to increase the bore diameter. External pressure on a piston acts to decrease its diameter. Hence, pressure acts to increase the clearance between a piston and its bore. Well-known thick-wall cylinder strain relationships[2] can be used to develop the following expressions for change in the diametral clearance due to a constant pressure in the clearance:

$$\Delta C = MP \quad (3)$$

where

$$M = \frac{D}{E}\left[\frac{2(D_0/D)^2 - 2(D_i/D)^2}{(D_0/D)^2 + (D_i/D)^2 - (D_0/D)^2(D_i/D)^2 - 1}\right] \quad (3a)$$

if the piston and cylinder are of the same material;

$$M = \frac{D}{E}\left[\frac{(D_0/D)^2 + 1}{(D_0/D)^2 - 1} + 1\right] \quad (3b)$$

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if the piston and cylinder are of the same material, and the piston is solid ($D_i = 0$);

$$M = D\left[\frac{1}{E_c}\left(\frac{(D_0/D)^2+1}{(D_0/D)^2-1}+\sigma_c\right)+\frac{1}{E_p}\left(\frac{1+(D_i/D)^2}{1-(D_i/D)^2}-\sigma_p\right)\right] \quad (3c)$$

if the piston and cylinder are of different materials;

$$M = D\left[\frac{1}{E_c}\left(\frac{(D_i/D)^2+1}{(D_0/D)^2-1}+\sigma_c\right)+\frac{1}{E_p}(1-\sigma_p)\right] \quad (3d)$$

if the piston and cylinder are of different materials, and the piston is solid ($D_i = 0$).

Diameters D_0, D_i, and D are defined on Fig. 2. E and σ are the modulus of elasticity and the Poisson ratio, respectively, for the piston (E_p, σ_p) and cylinder (E_c, σ_c) materials. M is a constant for a particular piston and cylinder. Figure 2 shows useful nondimensional curves from which diametral strains due to pressure, ΔD and ΔC, can be found for a wide range of materials, pressures, and physical configurations. Expressions (3) and Fig. 2 are applicable to uniform cylinders and pistons where end conditions are negligible.

In practice, pressure decreases along the clearance annulus from supply pressure at the leading end of the piston land to discharge pressure (often atmospheric). The shape of the pressure drop curve is dependent on the actual shape of the clearance space under operating conditions. The shape of the clearance passage can be affected by the attitude of the piston in the bore, by the strains induced in the components by pressure, and by temperature effects on the dimensions of the passage. The simplest model that can be assumed is that 1) the pressure drop along the piston land is linear and 2) the clearance annulus expands linearly according to Eq. (3) to correspond with the linear pressure drop, as illustrated on Fig. 3a.

This assumption is inadequate for the precise calculations of pressure distribution required for estimating lateral forces on the piston, or for the calculation of elastic deformations of the cylinder required to estimate the stresses in the cylinder. However, experimental measurements made over a wide range of clearance, reported later in this paper, show that this simple model is good for leakage flowrate calculations.

It can be shown that flow rate per unit width through a straight-sided convergently tapered passage of small depth (Fig. 3b) is given by

$$Q_x = \frac{P}{12\mu L}\cdot\frac{2h_1{}^2h_2{}^2}{h_1+h_2} = \frac{P}{12\mu L}\cdot h_e{}^3 \quad (4)$$

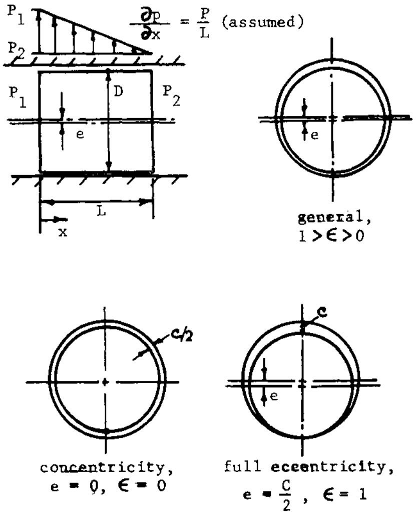


Fig. 1 Piston-cylinder configuration.

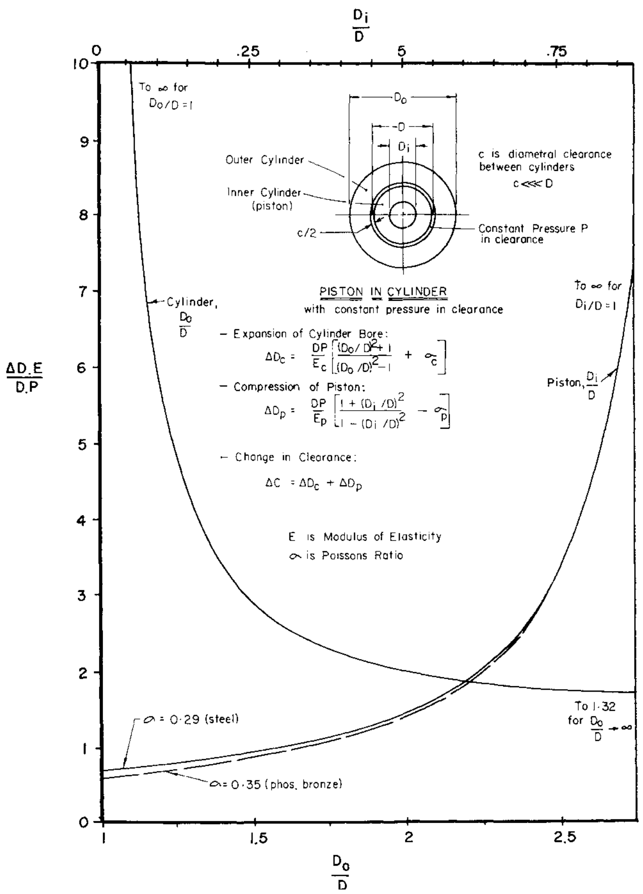


Fig. 2 Nondimensional curves for assessing the diametral strain of thick-wall cylinders due to internal and external pressures.

where h_1 and h_2 are the depths of the passage at inlet and discharge respectively; and $h_e = [2h_1{}^2h_2{}^2/(h_1 + h_2)]^{1/3}$ is the mean effective clearance. Using the proposed linear expansion model, this concept can be extended to the annular clearance of Fig. 3a to yield

$$C_e = \left[\frac{2(C+MP_1)^2(C+MP_2)^2}{(C+MP_1)+(C+MP_2)}\right]^{1/3} \quad (5)$$

where $C + MP_1$ corresponds to h_1, $C + MP_2$ corresponds to h_2, and M is calculated from Eq. (3).

Equation (5) provides the mean effective clearance C_e to be used in Eq. (2). It should be kept in mind that use of the eccentricity factor (ϵ) requires that the piston and cylinder bore axes remain parallel. To illustrate the importance of the correction, consider the following example: $D = 0.5$ in., $D_0 = 1.5$ in., $C = 3 \times 10^{-4}$ in., supply pressure $P_1 = 5000$ lb/in.^{2}, and discharge pressure P_2 is 0, so that $P = 5000$ lb/in.^{2}, $E = 30 \times 10^6$ lb/in.^{2}, $D_i = 0$, $E_p = E_c$, and $\sigma_p = \sigma_c$.

Fig. 3a Convergently tapered clearance.

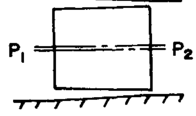
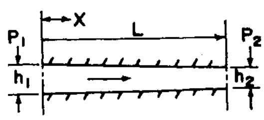


Fig. 3b Convergently tapered passage.



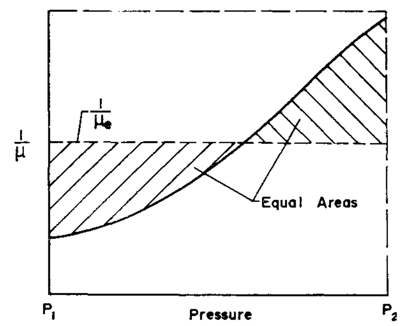


Fig. 4 Mean effective viscosity from viscosity-pressure drop curve.

At the high-pressure end of the piston, clearance is

$$C + \Delta C = C + MP_1 = \left[3 + \frac{0.5}{30 \times 10^6} \cdot \left(\frac{1.5^2 + 0.5^2}{1.5^2 - 0.5^2} + 1\right) \times 5000\right] 10^{-4}$$

$$= (3 + 1.875) \times 10^{-4} = 4.875 \times 10^{-4} \text{ in.}$$

At the discharge end of the piston, clearance is $C = 3 \times 10^{-4}$ in.;

$$\therefore C_e = \left[\frac{2 \times 4.875^2 \times 3^2}{4.875 + 3}\right]^{1/3} \times 10^{-4} = 3.795 \times 10^{-4} \text{ in.}$$

Now $(C_e/C)^3 = (3.795/3)^3 = 2$, which shows that the calculated flowrate when the allowance is made for pressure effects on the clearance is twice that obtained if this effect is ignored. In general, the smaller the nominal clearance, the more significant is the effect of pressure expansion of the clearance on the flowrate.

Mean Effective Viscosity

The viscosity of mineral oils is affected by both temperature and pressure. Viscosity decreases as temperature increases, and increases as pressure increases. For the present type of clearance flow, pressure drops along the piston land from supply pressure to discharge pressure, while oil temperature rises due to viscous shear. The mode of viscosity change during flow is complex and requires knowledge of both pressure and temperature distributions in the clearance passage.

Effect of Flow on Oil Temperature

The power loss due to leakage flow is PQ_x lb in./min. Assuming that all of the lost energy is absorbed into heating the oil, and taking into account the Joule-Thomson cooling effect which accompanies the change in internal energy of the oil due to isenthalpic expansion, the temperature rise in the oil due to pressure drop P during flow is[3]

$$T = \frac{P}{C_p w m}\left[1 - T\left(\frac{\partial v/v}{\partial T}\right)_P\right] \quad (6)$$

where C_p is the specific heat at constant pressure of the oil, w is the specific weight of the oil, m is the work-heat conversion factor, v is the specific volume of the oil, $[(\partial v/v)/\partial T]_P$ is the thermal coefficient of volumetric expansion of the oil at constant pressure.

It is of interest that Eq. (6) is independent of the geometrical dimensions of the components forming the flow passage. The piston and cylinder will be good conductors of heat, and some of the heat generated by the pressure drop and flow of the oil will pass through these components. Normally, the temperature rise in the oil will be less than is indicated by Eq. (6). However, Eq. (6) gives a useful upper limit to the degree of heating of the oil during leakage.

Example

The relevant data on Shell Tellus 27 nonadditive-type industrial hydraulic oil is: viscosity = 55 CP at 80° F; specific weight = 0.0314 lb/in.^{3} at 80° F; specific heat = 0.5 Btu/lb °F in the pressure range 0–5000 lb/in.^{2} and temperature range 60°–100°F; coefficient of expansion $[(\partial v/v)/\partial T]_P = 3.75 \times 10^{-4}$/°F in the pressure range 0–5000 lb/in^2 and temperature range 60–100°F; $m = 9.35 \times 10^3$ lb in./Btu. For this oil, Eq. (6) yields

$$T = [P/(0.5 \times 0.0314 \times 9.35 \times 10^3)] \times [1 - 540 \times 3.75 \times 10^{-4}]$$

$$= 5.5 \times 10^{-3} P \text{ °F} \quad \text{or} \quad 5.5 \text{ °F per 1000 lb/in.}^2 \text{ pressure drop} \quad (6a)$$

It is of interest that inclusion of the $T[(\partial v/v)/\partial T]_P$ term in Eq. (6) reduces the calculated temperature rise by 20% in the present instance.

For the leakage experiments reported later in this paper, both the supply oil and the ambient temperature in the vicinity of the rig were maintained at 75° F. It was found that the temperature of the oil rose during leakage according to the relationship

$$T = 3.5 \times 10^{-3} P \text{ °F} \quad (6b)$$

Thus, about 36% of the heat generated in the oil during expansion was transferred to the piston and cylinder components in this case.

Effect of Flow on Oil Viscosity

The following method for evaluating a mean effective viscosity for use in flow calculations allows for changes in viscosity during flow. For a particular piston-to-cylinder configuration, Eq. (1) can be written

$$Q_x = K/\mu \cdot dp/dx \quad (7)$$

where $K = (\pi D C^3/96)(1 + 1.5\epsilon^2)$ describes the configuration, and dp/dx is the slope of the axial pressure drop. For easy application, a form

$$Q_x = KP/\mu_e L \quad (8)$$

is required, where μ_e is the mean effective viscosity of the oil during leakage flow. From (7), $Q_x dx = K(1/\mu)\cdot dp$, and integrating,

$$Q_x L = K \int_{P_2}^{P_1} \frac{1}{\mu} \cdot dp$$

and

$$Q_x = \frac{KP}{L} \cdot \frac{1}{P} \int_{P_2}^{P_1} \frac{1}{\mu} \cdot dp = \frac{KP}{\mu_e L} \quad (9)$$

where

$$\frac{1}{\mu_e} = \frac{1}{P} \int_{P_2}^{P_1} \frac{1}{\mu} \cdot dp. \quad (10)$$

Now,

$$\frac{1}{P} \int_{P_2}^{P_1} \frac{1}{\mu} \cdot dp$$

represents the mean ordinate of a plot of $1/\mu$ against pressure, as shown in Fig. 4. Hence, the mean effective viscosity during flow past a piston can be calculated if the viscosity-pressure drop data is available.

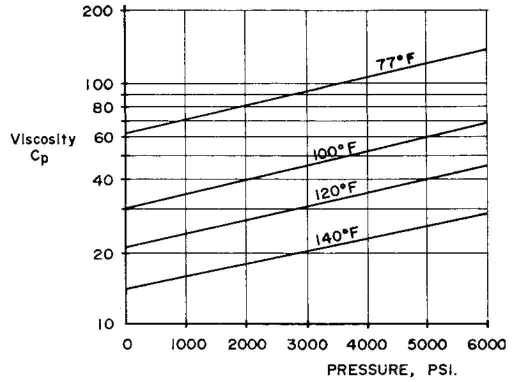


Fig. 5 Temperature-pressure-viscosity data for Shell Tellus 27 nonadditive mineral oil for hydraulic systems.

To Obtain Mean Effective Viscosity

1) The viscosity-pressure-temperature data for the oil used must be obtained. Figure 5 shows such data for Shell Tellus 27 oil.

2) The known pressure and temperature of the oil at the pressure end of the piston is used to obtain the initial viscosity (μ_1 at P_1, T_1) of the oil.

3) For a discrete portion ($P_1 - p$) of the total pressure drop ($P = P_1 - P_2$) along the piston, the temperature rise in the oil due to the leakage flow can be obtained from Eq. (6), modified if necessary to account for heat transfer through the components. Thus, the temperature and pressure at the point considered is known and the viscosity can be obtained from the oil data.

4) Repeat (3) for a series of pressure drops to cover the full pressure drop $P_1 - P_2$, getting the viscosity of the oil at various pressures in the pressure range.

5) Plot $1/\mu$ against P, and obtain the mean ordinate of the curve to yield $1/\mu_e$ and hence μ_e.

Example

Consider the oil whose pressure-temperature-viscosity characteristics are given on Fig. 5, and a leakage condition where $P_1 = 4000$ psi, $P_2 = 0$ psi, $T_1 = 77°$F, and heat transfer through the components is negligible so that Eq. (6b) is applicable. Table 1 shows the temperature and the viscosity of the oil at pressure drop increments of 800 psi. Figure 6 shows a plot of $1/\mu$ against pressure in the clearance. The area under the curve can be measured and the mean ordinate calculated from

$$\frac{1}{\mu_e} = \frac{\text{area under curve}}{\text{length of curve along the pressure axis}}$$

to give a mean effective viscosity of $\mu_e = 52.5$ CP.

It might be noted that μ_e is approximately equal to the value of viscosity at zero pressure and inlet temperature. This empirical approximation is useful with straight mineral oils for relatively low pressure and restricted temperature situations. However, the method presented here is fundamentally sound and is applicable to all fluids, pressure ranges, and temperatures. The mean effective viscosity μ_e, estimated with the foregoing procedures, is used in Eq. (2).

Table 1 Viscosity variation during leakage flow[a]

Pressure, lb/in.²	P_1 = 4000	3200	2400	1600	800	P_2 = 0
Temperature, °F	T_1 = 77	81.4	85.8	90.2	94.6	99
Viscosity, CP	96	76.5	60	49	38.5	31

[a] Viscosity at $P_1 = 0$ lb/in.² and $T_1 = 77°$F. is 54 CP

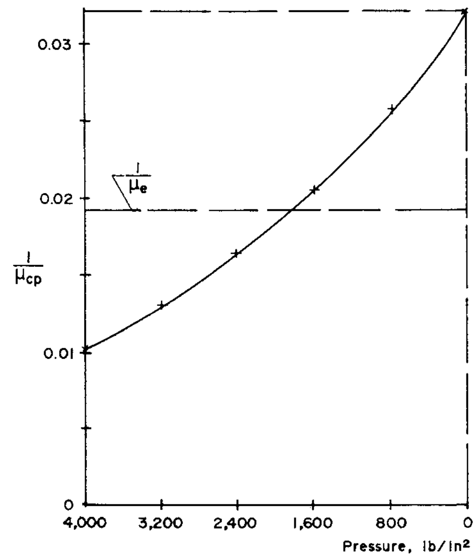


Fig. 6 Variation of $1/\mu$ along a clearance passage.

Experimental Varification of Leakage Equation

Leakage flowrate measurements were made on an experimental rig assembled for hydraulic lock experiments.‡ Briefly, the test feature was a pair of single-land pistons set back-to-back against a common reaction member. Oil in the system was cleaned by continuous circulation through fine filters. The level of cleanliness attained was such that no measurable decrease in flowrate (due to silting in the clearance) would be observed when leakage measurements were made over periods of up to 1 hr at constant applied pressure and with the pistons stationary. Piston land length and diameter were 1 in. and 0.5 in., respectively. Pistons having different clearances could be fitted to the bores. The magnitude of the hydraulic lock present during the leakage rate tests indicated that the pistons lay against the bore, very close to the parallel axes condition $\epsilon = 1$, throughout the tests.

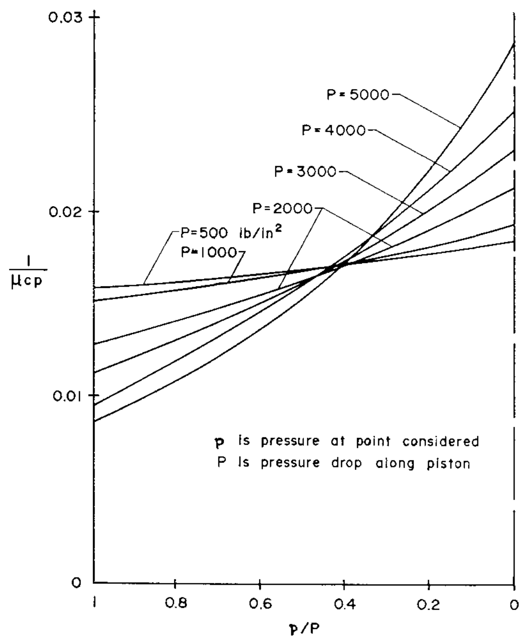


Fig. 7 Calculated viscosity distribution during leakage at several pressure drops (Shell T27 oil).

‡ The subject of a paper accepted for publication in the *Proceedings of the Institution of Mechanical Engineers*, London.

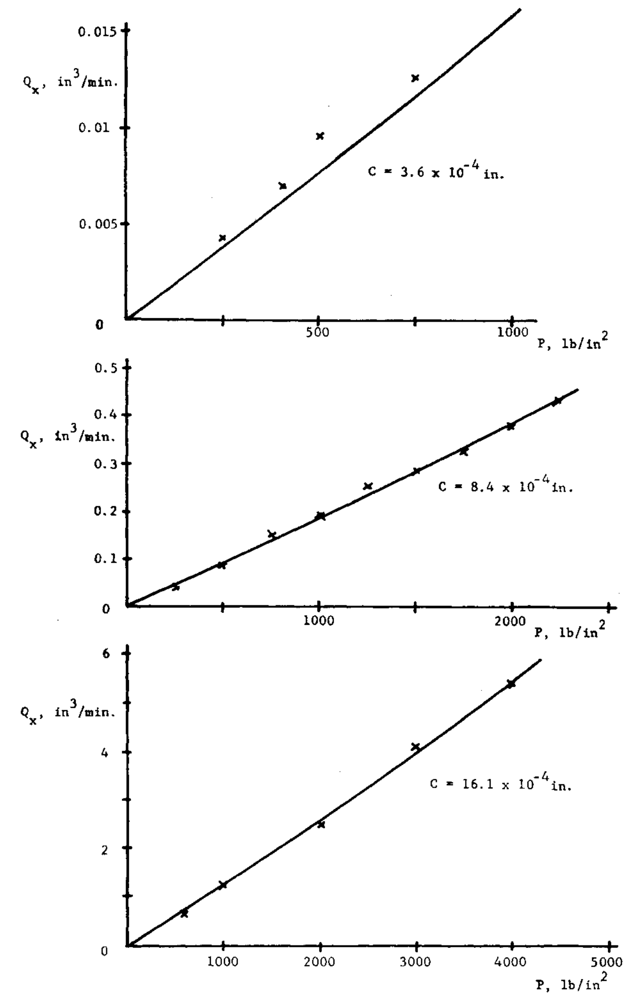


Fig. 8. Experimental (data points) and calculated (lines) leakage flowrates. Piston length = 1 in., diameter = 0.5 in.

Figure 7 shows the calculated viscosity-pressure drop data for several pressure drops. Equation (6b) was used for calculating temperature rises. The mean ordinate for each curve was taken and used to obtain the mean effect viscosities, which are given in Table 2. The results show that the mean effective viscosity increases somewhat with pressure drop, and that the range of viscosity occurring during leakage flow increases sharply with increasing pressure drop.

The effective clearance between piston and bore at each pressure drop was calculated for each of the three clearances used in the experiments, using Eq. (5). As the discharge pressure was zero, Eq. (5) reduced to

$$C = 2C^2(C + MP)^2/(2C + MP)$$

M was calculated from Eq. (3b) using $D = 0.5$ in., $D_0 = 1.5$ in., and $E = 30 \times 10^6$ lb/in.² Values of C_e are shown in Table 2.

Table 2 Effective viscosity and clearance at various pressure drops[a]

Calculated values of effective viscosity μ_e and clearance C_e	Pressure drop along piston land, lb/in.² 500	1000	2000	3000	4000	5000
μ_e, all clearances CP	57.5	58	59	59.5	59.5	60
C_e 10^{-4} in. C = 3.6 $\times$ 10^{-4} in.	3.7	3.82	3.96	4.12	4.26	4.4
C = 8.4 $\times$ 10^{-4} in.	8.5	8.6	8.78	9.0	9.12	9.2
C = 16.1 $\times$ 10^{-4} in.	16.2	16.3	16.52	16.7	16.88	17.1

[a] Viscosity at $P = 0$, $T = 75°F$ is 56.5 CP.

The data points of Fig. 8 are measured flowrates for the three diametral clearances. The lines shown are the flowrates calculated from Eq. (2) for the condition $\epsilon = 1$. The good agreement obtained for the wide range of clearances used indicates the effectiveness of Eq. (2). The leakage measurements presented are confined to the pressure drop regions in which it was known that the pistons were in contact with the bore, and the piston and bore axes were close to being parallel.

Conclusion

1) The basic equation for leakage flowrate through the clearance passage between piston-cylinder components [Eq. (1)] can be modified to include allowances for the effect of pressure on both the dimensions of the clearance, and on the viscosity of the oil during flow [Eq. (2)].

2) The basic flow relationship gives erroneous results for situations of high pressure drop and fine clearance if these allowances are not made.

3) A mean effective clearance can be calculated on the basis of the radial strain on the piston and bore due to pressure [Eq. (5)].

4) A mean effective viscosity can be assessed fundamentally by calculating the temperature rise in the oil which accompanies the pressure drop across the leakage path [Eq. (6), modified if necessary to account for heat transfer from the oil to the piston and cylinder components], and using the temperature-pressure-viscosity data for the oil in the manner described by Eq. (10) and illustrated in Fig. 4.

5) Experimental measurements made with pistons having a wide range of diametral clearance (3×10^{-4} to 16×10^{-4} in.) confirm the validity of Eq. (2).

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